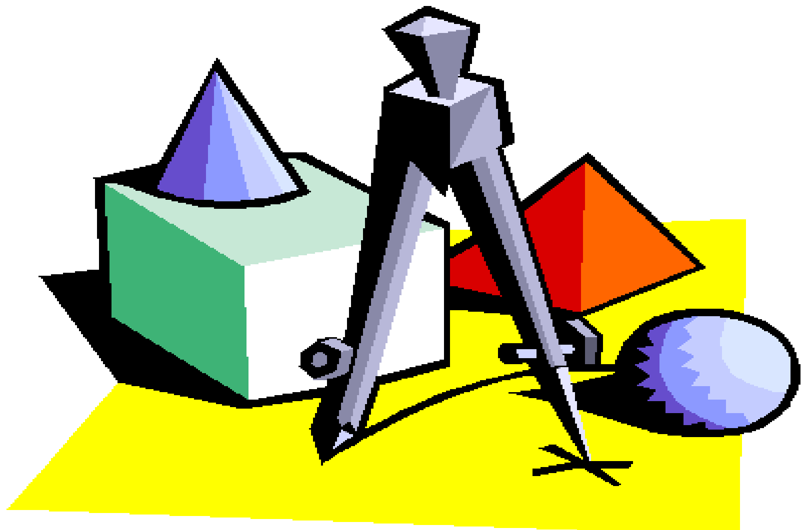
Geometry



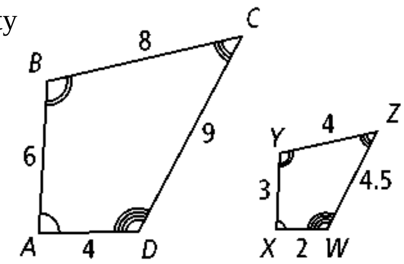
Day 1	Day 2	Day 3	Day 4	Day 5
Topic 9 Lesson 1 Similar Polygons	Topic 9 Lesson 1 Similar Polygons	Topic 9 Lesson 4 Similarity in Right Triangles	Topic 9 Lesson 4 Similarity in Right Triangles	Topic 9 Lesson 5 Proportions in Triangles
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
Topic 9 Lesson 5 Proportions in Triangles	Topic 10 Lesson 1 Pythagorean Theorem and its converse	Topic 10 Lesson 1 Pythagorean Theorem and its converse	Topic 10 Lesson 2 Special Right Triangles	Topic 10 Lesson 2 Special Right Triangles

Name_____________________________________Class______________Date_____________

9-1 Reteaching

Similar polygons have corresponding angles that are congruent and corresponding sides that are proportional. A number of proportions can be written for the ratios of corresponding sides of similar polygons.

Are the quadrilaterals at the right similar? If so, write a similarity statement and an extended proportion.



Compare angles: $\angle A \cong \angle X$, $\angle B \cong \angle Y$.

$\angle C \cong \angle Z$, $\angle D \cong \angle W$

Compare ratios of sides: $\dfrac{AB}{XY} = \dfrac{6}{3} = 2$ $\quad \dfrac{CD}{ZW} = \dfrac{9}{4.5} = 2$

$\dfrac{BC}{YZ} = \dfrac{8}{4} = 2$ $\quad \dfrac{DA}{WX} = \dfrac{4}{2} = 2$

Because corresponding sides are proportional and corresponding angles are congruent, $ABCD \sim XYZW$.

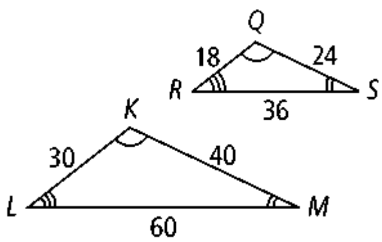
The proportions for the ratios of corresponding sides are:

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZW} = \frac{DA}{WX}$$

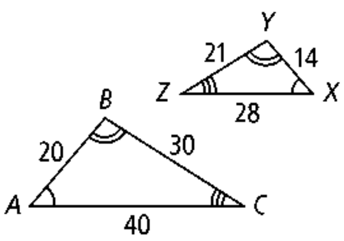
Exercises

If the polygons are similar, write a similarity statement and the extended proportion for the ratios of corresponding sides. If the polygons are not similar, write *not similar.*

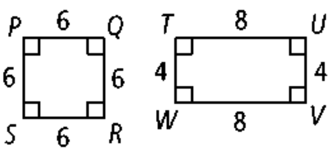
1.



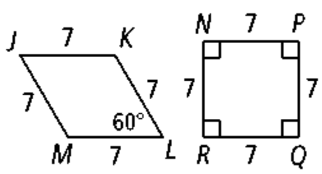
2.



3.



4.



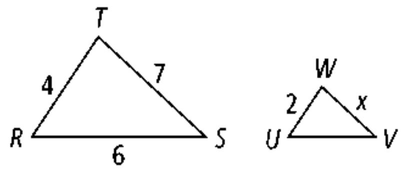
Name______________________________________ Class______________ Date_____________

9-1 Reteaching (continued)

Problem

$\Delta RST \sim \Delta UVW$. What is the scale factor?

What is the value of x?



Identify corresponding sides: $\overline{RT}$ corresponds to $\overline{UW}$, $\overline{TS}$ corresponds to $\overline{WV}$, and $\overline{SR}$ corresponds to $\overline{VU}$.

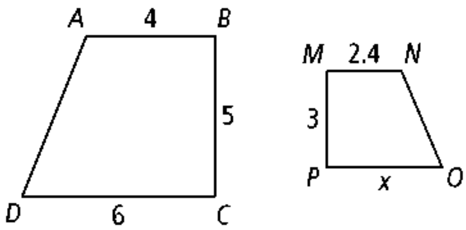
$\frac{RT}{UW} = \frac{TS}{WV}$	Compare corresponding sides.
$\frac{4}{2} = \frac{7}{x}$	Substitute.
$4x = 14$	Cross Products Property
$x = 3.5$	Divide each side by 4.

The scale factor is $\frac{4}{2} = \frac{7}{3.5} = 2.$ The value of x is 3.5.

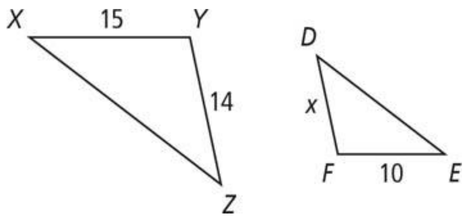
Exercises

Give the scale factor of the polygons. Find the value of *x*. Round answers to the nearest tenth when necessary.

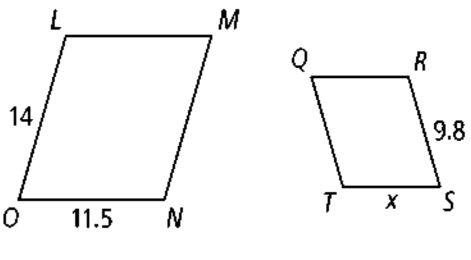
5. $ABCD \sim NMPO$



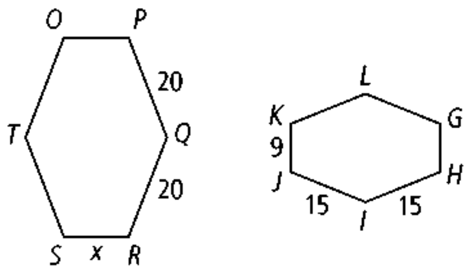
6. ΔXYZ, ΔEFD



7. $LMNO \sim RQTS$



8. $OPQRST \sim GHIJKL$



Name ______________________________ Class ____________ Date ____________

9-2 Reteaching

Similarity Transformations

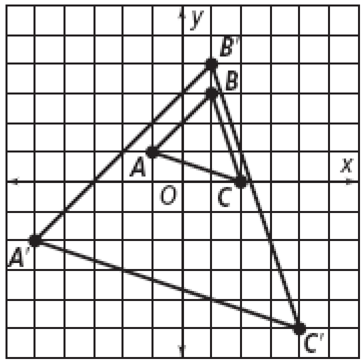
Dilations and compositions of dilations and rigid motions form a special class of transformations called *similarity transformations*. Similarity can be defined by similarity transformations as follows.

Two figures are *similar* if and only if there is a similarity transformation that maps one figure to the other.

Thus, if you have a similarity transformation, the image of the transformation is similar to the preimage. Use the rules for each transformation in the composition to find the image of a point in the transformation.

Problem

ΔABC has vertices $A(-1, 1)$, $B(1, 3)$, and $C(2, 0)$. What is the image of ΔABC when you apply the similarity transformation $T_{<-2,\,-5>} \circ D_3$?



For any point (x, y), $D_3(x, y) = (3x, 3y)$ and $T_{<-2,\,-5>}(x, y) = (x - 2, y - 5)$.

$(T_{<-2,\,-5>} \circ D_3)(A) = T_{<-2,\,-5>}\,(D_3(A)) =$
$T_{<-2,\,-5>}(-3, 3) = (-5, -2)$

$T_{<-2,\,-5>} \circ D_3)(B) = T_{<-2,\,-5>}\,D_3(B))$
$= T_{<-2,\,-5>}(3, 9) = (1, 4)$

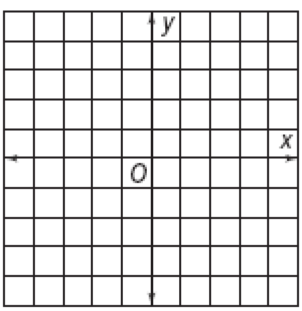
$(T_{<-2,\,-5>} \circ D_3)(C) = T_{<-2,\,-5>}(D_3(C)) = T_{<-2,\,-5>}\,(6, 0) = (4, -5)$

Thus, the image has vertices $A'(-5, -2)$, $B'(1, 4)$, and $C'(4, -5)$ and is similar to ΔABC.

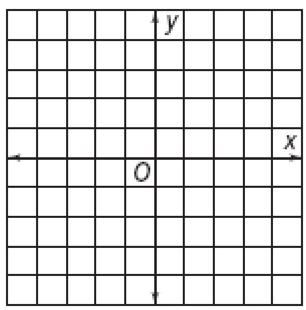
Exercises

ΔABC has vertices $A(-2, 1)$, $B(-1, -2)$, and $C(2, 2)$. For each similarity transformation, draw the image.

1. $D_2 \circ R_{y\text{-axis}}$



2. $r_{(90°,\,O)} \circ D_2$



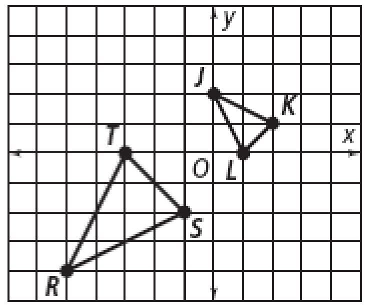
9-2 Reteaching

Similarity Transformations

If you can show that a similarity transformation maps one figure to another, then you have shown that the two figures are similar.

Problem

Show that $\Delta JKL \sim \Delta RST$ by finding a similarity transformation that maps one triangle to the other.



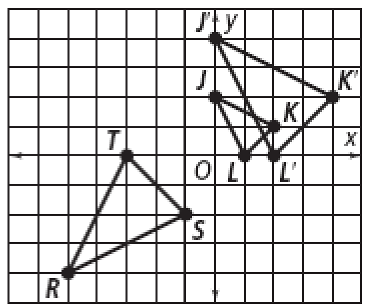
ΔRST appears to be twice the size of ΔJKL, so dilate by scale factor of 2. Map each vertex using D_2.

$D_2(J) = (0, 4) = J'$

$D_2(K) = (4, 2) = K'$

$D_2(L) = (2, 0) = L'$

Graph the image of the dilation $\Delta J'K'L'$.



$\Delta J'K'L'$ is congruent to ΔRST and can be mapped to ΔRST by the glide reflection $R_{x\text{-axis}} \circ T_{<-5,\,0>}$.

Verify that each vertex of $\Delta J'K'L'$ maps to a vertex of ΔRST.

$(R_{x\text{-axis}} \circ T_{<-5,\,0>})(J') = R_{x\text{-axis}}(-5, 4) = (-5, -4) = R$

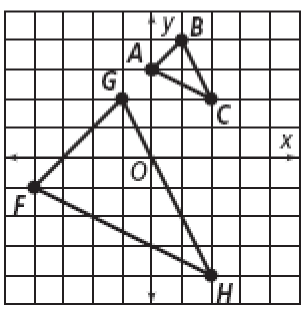
$(R_{x\text{-axis}} \circ T_{<-5,\,0>})(K') = R_{x\text{-axis}}(-1, 2) = (-1, -2) = S$

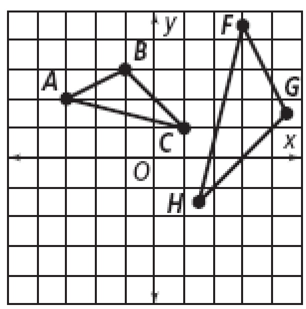
$(R_{x\text{-axis}} \circ T_{<-5,\,0>})(L') = R_{x\text{-axis}}(-3, 0) = (-3, -0) = T$

Thus, the similarity transformation $R_{x\text{-axis}} \circ T_{<-5,\,0>} \circ D_2$ maps ΔJKL to ΔRST.

Exercises

For each pair of figures, find a similarity transformation that maps ΔABC to ΔFGH. Then, write the similarity statement.

3. 

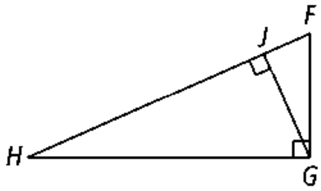
4. 

Name_____________________________________Class______________ Date_____________

9-4 Reteaching

Theorem 9-3

If you draw an altitude from the right angle to the hypotenuse of a right triangle, you create three similar triangles. This is Theorem 9-3. ΔFGH is a right triangle with right $\angle FGH$ and the altitude of the hypotenuse $\overline{JG}$. The two triangles formed by the altitude are similar to each other and similar to the original triangle.
So, $\Delta FGH \sim \Delta FJG \sim \Delta GJH$.



Two corollaries to Theorem 9-3 relate the parts of the triangles formed by the altitude of the hypotenuse to each other by their geometric mean.

The geometric mean, *x*, of any two positive numbers *a* and *b* can be found with the proportion $\frac{a}{x} = \frac{x}{b}$.

What is the geometric mean of 8 and 12?

$$\frac{8}{x} = \frac{x}{12}$$

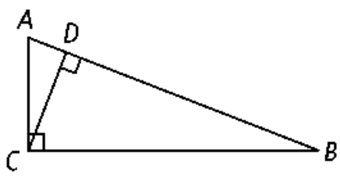
$$x^2 = 96$$

$$x = \sqrt{96} = \sqrt{16 \cdot 6} = 4\sqrt{6}$$

The geometric mean of 8 and 12 is $4\sqrt{6}$.

Corollary 1 to Theorem 9-3

The altitude of the hypotenuse of a right triangle divides the hypotenuse into two segments. The length of the altitude is the geometric mean of these segments.



Since $\overline{CD}$ is the altitude of right ΔABC, it is the geometric mean of the segments of the hypotenuse AD and DB:

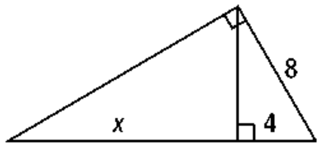
$$\frac{AD}{CD} = \frac{CD}{DB}.$$

Name____Class_______________ Date______________

9-4 Reteaching (continued)

Corollary 2 to Theorem 9-3

The altitude of the hypotenuse of a right triangle divides the hypotenuse into two segments. The length of each leg of the original right triangle is the geometric mean of the length of the entire hypotenuse and the segment of the hypotenuse adjacent to the leg. To find the value of *x*, you can write a proportion.

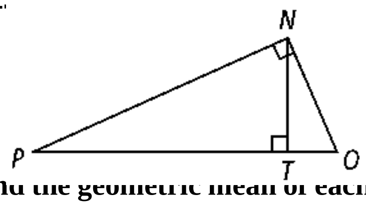


$\dfrac{\text{segment of hypotenuse}}{\text{adjacenleg}} = \dfrac{\text{adjacent leg}}{\text{hypotenuse}} \quad \dfrac{4}{8} = \dfrac{8}{4+x}$	Corollary 2
$4(4 + x) = 64$	Cross Products Property
$16 + 4x = 64$	Simplify.
$4x = 48$	Subtract 16 from each side.
$x = 12$	Divide each side by 4.

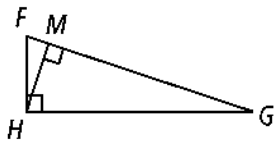
Exercises

Write a similarity statement relating the three triangles in the diagram.

1.



2.

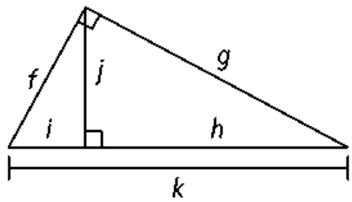


Find the geometric mean of each pair of numbers.

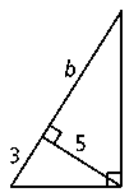
3. 2 and 8 **4.** 4 and 6 **5.** 8 and 10 **6.** 25 and 4

Use the figure to complete each proportion.

7. $\dfrac{i}{\square} = \dfrac{f}{k}$ **8.** $\dfrac{i}{j} = \dfrac{j}{\square}$ **9.** $\dfrac{\square}{f} = \dfrac{f}{\square}$



10. A classmate writes the proportion $\dfrac{3}{5} = \dfrac{5}{(3+b)}$ to find *b*. Explain why the proportion is incorrect and provide the right answer.

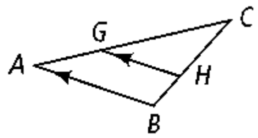


Name_____________________________________Class______________ Date_____________

9-5 Reteaching

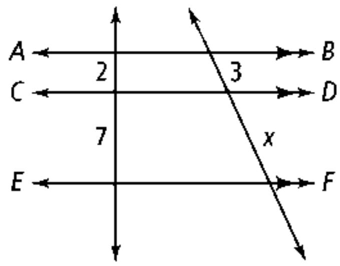
The Side-Splitter Theorem states the proportional relationship in a triangle in which a line is parallel to one side while intersecting the other two sides.

Theorem 9-4: Side-Splitter Theorem



In $\triangle ABC$, $\overline{GH} \parallel \overline{AB}$. $\overline{GH}$ intersects $\overline{BC}$ and $\overline{AC}$. The segments of $\overline{BC}$ and $\overline{AC}$ are proportional: $\dfrac{AG}{GC} = \dfrac{BH}{HC}$

The corollary to the Side-Splitter Theorem extends the proportion to three parallel lines intercepted by two transversals.
If $\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$, you can find x using the proportion:



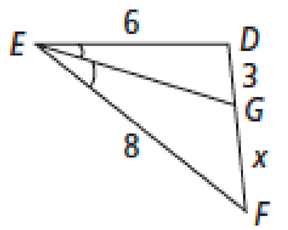
$$\frac{2}{7} = \frac{3}{x}$$

$2x = 21$ Cross Products Property

$x = 10.5$ Solve for *x*.

Theorem 9-5: Triangle-Angle-Bisector Theorem

When a ray bisects the angle of a triangle, it divides the opposite side into two segments that are proportional to the other two sides of the triangle.



In $\triangle DEF$, $\overline{EG}$ bisects $\angle E$. The lengths of $\overline{DG}$ and $\overline{DF}$ are proportional to their adjacent sides $\overline{DF}$ and $\overline{EF}$: $\dfrac{DG}{DE} = \dfrac{GF}{EF}$.

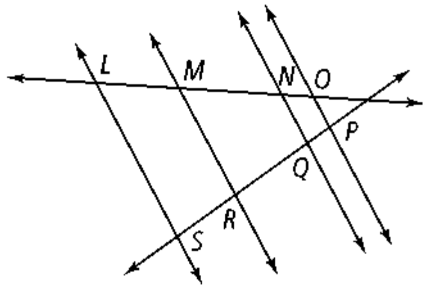
To find the value of x, use the proportion $\dfrac{3}{6} = \dfrac{x}{8}$.

$$6x = 24$$

$$x = 4$$

Exercises

Use the figure at the right to complete each proportion.



1. $\dfrac{\boxed{}}{MN} = \dfrac{SR}{LM}$

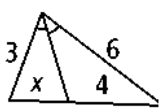
2. $\dfrac{NO}{\boxed{}} = \dfrac{LM}{SR}$

3. $\dfrac{MN}{RQ} = \dfrac{\boxed{}}{QP}$

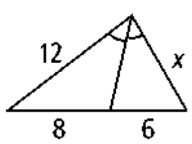
4. $\dfrac{SQ}{LN} = \dfrac{RP}{\boxed{}}$

Solve for *x*.

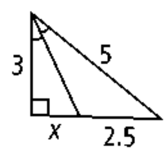
5.



6.



7.

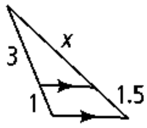


Name___________________________________ Class_____________ Date_____________

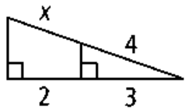
9-5 Reteaching (continued)

Solve for *x*.

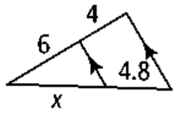
8.



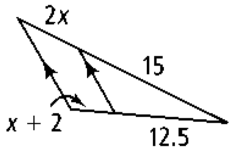
9.



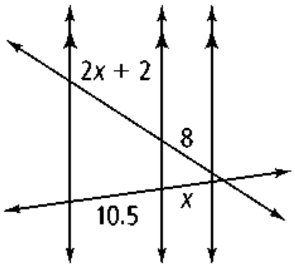
10.



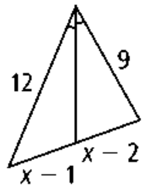
11.



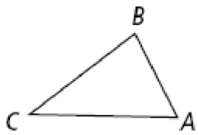
12.



13.



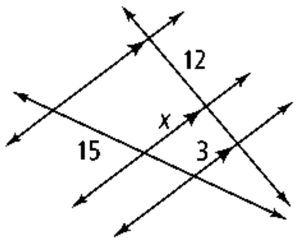
In △*ABC*, *AB* = 6, *BC* = 8, and *AC* = 9.



14. The bisector of $\angle A$ meets $\overline{BC}$ at point *N*. Find *BN* and *CN*.

15. $\overline{XY} \parallel \overline{CA}$. Point *X* lies on $\overline{BC}$ such that *BX* = 2, and *Y* is on $\overline{BA}$. Find *BY*.

16. A classmate says you can use the Corollary to the Side-Splitter Theorem to find the value of *x*. Explain what is wrong with your classmate's statement.



17. An angle bisector of a triangle divides the opposite side of the triangle into segments 6 and 4 in. long. The side of the triangle adjacent to the 6-in. segment is 9 in. long. How long is the third side of the triangle?

18. ΔGHI has angle bisector $\overline{GM}$, and *M* is a point on $\overline{HI}$. *GH* = 4, *HM* = 2, *GI* = 9. Solve for *MI*. Use a drawing to help you find the answer.

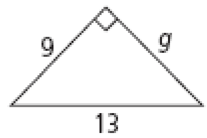
19. The lengths of the sides of a triangle are 7 mm, 24 mm, and 25 mm. Find the lengths to the nearest tenth of the segments into which the bisector of each angle divides the opposite side.

Name ______________________________ Class ____________ Date ____________

10-1 Reteaching

The Pythagorean Theorem and Its Converse

The *Pythagorean Theorem* can be used to find the length of a side of a right triangle.



Pythagorean Theorem: $a^2 + b^2 = c^2$, where a and b are the legs of a right triangle, and c is the hypotenuse.

Problem

What is the value of g? Leave your answer in simplest radical form.

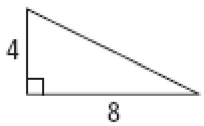
Using the Pythagorean Theorem, substitute g and 9 for the legs and 13 for the hypotenuse.

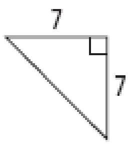
$a^2 + b^2 = c^2$	
$g^2 + 9^2 = 13^2$	Substitute.
$g^2 + 81 = 169$	Simplify.
$g^2 = 88$	Subtract 81 from each side.
$g = \sqrt{88}$	Take the square root.
$g = \sqrt{4(22)}$	Simplify.
$g = 2\sqrt{22}$	

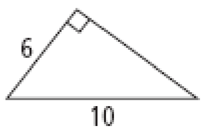
The length of the leg, g, is $2\sqrt{22}$.

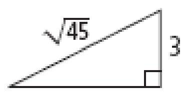
Exercises

Identify the values of *a*, *b*, and *c*. Write ? For unknown values. Then, find the missing side lengths. Leave your answers in simplest radical form.

1. 

2. 

3. 

4. 

5. A square has side length 9 in. What is the length of the longest line segment that can be drawn between any two points of the square?

6. Right ΔABC has legs of lengths 4 ft and 7 ft. What is the length of the triangle's hypotenuse?

7. Televisions are sold by the length of the diagonal across the screen. If a new 48-in. television screen is 42 in. wide, how tall is the screen to the nearest inch?

10-1 Reteaching (continued)

The Pythagorean Theorem and Its Converse

Use Theorems 8-3 and 8-4 to determine whether a triangle is acute or obtuse.

Let a and b represent the shorter sides of a triangle and c represent the longest side.

If $a^2 + b^2 > c^2$, then the triangle is acute

If $a^2 + b^2 < c^2$, then the triangle is obtuse.

Problem

A triangle has side lengths 6, 8, and 11. Is the triangle *acute, obtuse,* or *right?*

$a = 6, b = 8, c = 11$	Identify *a, b,* and *c.*
$a^2 + b^2 = 6^2 + 8^2$	Substitute to find $a^2 + b^2$.
$a^2 + b^2 = 36 + 64$	
$a^2 + b^2 = 100$	
$c^2 = 11^2$	Substitute to find c^2.
$c^2 = 121$	
$100 < 121$	Compare $a^2 + b^2$ and c^2.

$a^2 + b^2 < c^2$, so the triangle is obtuse.

Exercises

The lengths of the sides of a triangle are given. Classify each triangle as *acute, right,* or *obtuse.*

8. 7, 9, 10

9. 18, 16, 24

10. 3, 5, $5\sqrt{2}$

11. 10, 10, $10\sqrt{2}$

12. 8, 6, 10.5

13. 7, $7\sqrt{3}$, 14

14. 22, 13, 23

15. 17, 19, 26

16. 21, 28, 35

17. A local park in the shape of a triangle is being redesigned. The fencing around the park is made of three sections. The lengths of the sections of fence are 27 m, 36 m, and 46 m. The designer of the park says that this triangle is a right triangle. Is he correct? Explain.

18. Your neighbor’s yard is in the shape of a triangle, with dimensions 120 ft, 84 ft, and 85 ft. Is the yard an *acute, obtuse,* or a *right* triangle? Explain.

Name ______________________________________ Class _____________ Date ______________

10-2 Reteaching

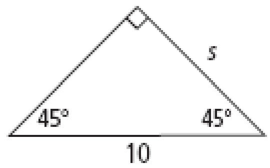
Special Right Triangles

In a 45°-45°-90° triangle, the legs are the same length.

$$\text{hypotenuse} = \sqrt{2} \times \text{leg}$$

Problem

What is the value of the variable, *s*?



$10 = s\sqrt{2}$	In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times the length of the leg.
$s = \frac{10}{\sqrt{2}}$	Divide both sides by $\sqrt{2}$.
$\frac{10}{\sqrt{2}}\, g\, \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$	Rationalize the denominator.
$s = 5\sqrt{2}$	

In a 45°-45°-90° triangle the length of the leg is $\frac{\sqrt{2}}{2} \times$ hypotenuse.

Exercises

Complete each exercise.

1. Draw a horizontal line segment on centimeter grid paper so that the endpoints are at the intersections of grid lines.

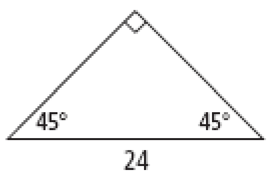
2. Use a protractor and a straightedge to construct a 45°-45°-90° triangle.

3. Use the 45°-45°-90° Triangle Theorem to calculate the lengths of the legs. Round to the nearest tenth.

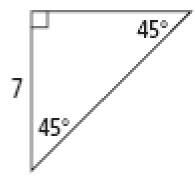
4. Measure the lengths of the legs to the nearest tenth of a centimeter. Compare your calculated results and your measured results.

Use the diagrams below each exercise to complete Exercises 5–7.

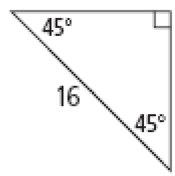
5. Find the length of the leg of the triangle.



6. Find the length of the hypotenuse of the triangle.



7. Find the length of the leg of the triangle.



10-2 Reteaching (continued)

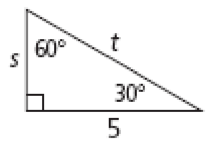
Special Right Triangles

In a 30°-60°-90° triangle, the longer leg is opposite the 60° angle and the shorter leg is opposite the 30° angle.

$$\text{longer leg} = \sqrt{3} \times \text{short leg}$$
$$\text{hypotenuse} = 2 \times \text{short leg}$$

Problem

Find the value of each variable.



$5 = \sqrt{3}s$ In a 30°-60°-90° triangle the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

$\frac{5}{\sqrt{3}} = s$ Divide both side by $\sqrt{3}$.

$s = \frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$ Rationalize the denominator.

The length of the hypotenuse is twice the length of the shorter leg.

$$t = 2\left(\frac{5\sqrt{3}}{3}\right) = \frac{10\sqrt{3}}{3}$$

Exercises

Complete each exercise.

8. Draw a horizontal line segment on centimeter grid paper so that the endpoints are at the intersections of grid lines.

9. Use a protractor and a straightedge to construct a 30°-60°-90° triangle with your segment as one of its sides.

10. Use the 30°-60°-90° Triangle Theorem to calculate the lengths of the other two sides. Round to the nearest tenth.

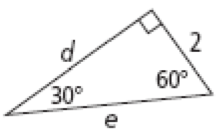
11. Measure the lengths of the sides to the nearest tenth of a centimeter.

12. Compare your calculated results with your measured results.

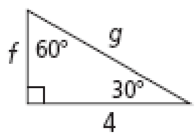
13. Repeat the activity with a different segment.

For Exercises 14–17, find the value of each variable.

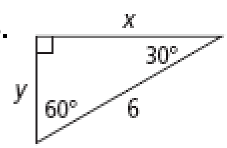
14.



15.



16.



17.